

Score	Evaluator	Section III: Work out math questions. (6 questions; 8 points for each; 48 points in total.)

1. The first question $\int e^{2x} (\tan x + 1)^2 dx$.

Solution. $I = \int e^{2x} \sec^2 x dx + 2 \int e^{2x} \tan x dx$ 2 points
 $= \int e^{2x} d(\tan x) + 2 \int e^{2x} \tan x dx$ 4 points
 $= e^{2x} \tan x - 2 \int e^{2x} \tan x dx + 2 \int e^{2x} \tan x dx$ 6 points
 $= e^{2x} \tan x + C$ 8 points

2. The second question $A(1, 2, -1), B(2, 3, 0), C(3, 3, 2)$ text $\triangle ABC$ text text text text text.

Solution. Text $\overrightarrow{AB} = (1, 1, 1), \overrightarrow{AC} = (2, 1, 3)$,2 points
 $\text{text } \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = (2, -1, -1),$ 4 points
 $\text{text } \triangle ABC \text{ text } S_{\triangle ABC} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2} \sqrt{6}.$ 6 points
 Text text $2(x-2) - (y-3) - z = 0$, text $2x - y - z - 1 = 0.$ 8 points

3. The third question $A = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix}$ text.

Solution. $A = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 0 & -1 & -6 & 1 \\ 0 & -6 & -8 & 2 \end{vmatrix} = 1 \cdot (-1)^{2+1} \begin{vmatrix} 1 & 2 & 3 \\ -1 & -6 & 1 \\ -6 & -8 & 2 \end{vmatrix}$ 4 points

$= - \begin{vmatrix} 1 & 2 & 3 \\ 0 & -4 & 4 \\ 0 & 4 & 20 \end{vmatrix} = - \begin{vmatrix} -4 & 4 \\ 4 & 20 \end{vmatrix} = -(-4 \cdot 20 - 4 \cdot 4) = 96$ 8 points

4. The fourth question, text text $f = x_1^2 + 2x_1x_2 - 6x_1x_3 + 2x_2^2 - 12x_2x_3 + 9x_3^2$ text text $f = d_1y_1^2 + d_2y_2^2 + d_3y_3^2$.

Solution. $f = x_1^2 + 2x_1x_2 - 6x_1x_3 + 2x_2^2 - 12x_2x_3 + 9x_3^2$
 $= x_1^2 + 2x_1(x_2 - 3x_3) + (x_2 - 3x_3)^2 + x_2^2 - 6x_2x_3$
 $= (x_1 + x_2 - 3x_3)^2 + x_2^2 - 6x_2x_3$ 3 points

$= (x_1 + x_2 - 3x_3)^2 + x_2^2 - 2x_2 \cdot 3x_3 + (3x_3)^2 - 9x_3^2$
 $= (x_1 + x_2 - 3x_3)^2 + (x_2 - 3x_3)^2 - 9x_3^2$ 6 points

Text $y_1 = x_1 + x_2 - 3x_3, y_2 = x_2 - 3x_3, y_3 = x_3,$
 text $f = y_1^2 + y_2^2 - 9y_3^2$ text.8 points

5. The fifth question text text text 0.2 text text, text text 100 text text.

(1) text text text text text text ξ text 10 text 30 text.

(2) text text text text text text ξ text 10 text 30 text.

Solution. $E\xi = np = 100 \cdot 0.2 = 20$, $D\xi = npq = 100 \cdot 0.2 \cdot 0.8 = 16$2 points

(1) $P(10 < \xi < 30) = P(|\xi - E\xi| < 10) \geq 1 - \frac{D\xi}{10^2} = 1 - \frac{16}{100} = 0.84$4 points

(2) $P(10 < \xi < 30) \approx \Phi_0\left(\frac{30-20}{\sqrt{16}}\right) - \Phi_0\left(\frac{10-20}{\sqrt{16}}\right)$ 6 points

$= 2\Phi_0(2.5) - 1 = 2 \cdot 0.9938 - 1 = 0.9876$ 8 points

6. The sixth question $N(\mu, \sigma^2)$ text text 16 text, text text text 3160, text text 100. Text text $H_0 : \mu = 3140$ text text ($\alpha = 0.01$).

Solution. (1) Text text $H_0 : \mu = 3140$2 points

(2) Text text text $T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1)$3 points

(3) Text text $t_\alpha = t_\alpha(n-1) = t_{0.01}(15) = 2.947$5 points

(4) Text text text $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{3160 - 3140}{100/4} = 0.8$7 points

(5) Text $|t| < t_\alpha$, text text H_0 , text text text.8 points

Score	Evaluator	Section IV: Work out math proofs. (2 questions; 16 points in total.)

1. (9 points) The first question $\{x_n\}$ text $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{2 + x_n}$. Text text text, text text text.

Proof. (1) Text, text $x_1 < 2$, text $x_k < 2$ text

$$x_{k+1} = \sqrt{2 + x_k} < \sqrt{2 + 2} = 2,$$

Text text text text text n text $x_n < 2$, text text text. Text text

$$\frac{x_{n+1}}{x_n} = \sqrt{\frac{2}{x_n^2} + \frac{1}{x_n}} > \sqrt{\frac{2}{2^2} + \frac{1}{2}} = 1,$$

Text text text text text. Text text text text text, Text text text text.4 points

(2) Text text text text A , text text text text text text

$$A = \sqrt{2 + A}.$$

Text text $A = 2$, text text $\{x_n\}$ text text text 2.8 points

2. (7 points) The second question A text B text, text A text \bar{B} text.

Proof. $P(A \cdot \bar{B}) = P(A - B) = P(A - AB)$ 2 points

$$= P(A) - P(AB) = P(A) - P(A)P(B)$$
4 points

$$= P(A)(1 - P(B)) = P(A)P(\bar{B})$$
6 points

Text text text A text text text \bar{B} text text text.8 points

Appendix Some data may be used in the exam

$\Phi_0(0.5) = 0.6915$	$\Phi_0(1) = 0.8413$	$\Phi_0(2) = 0.9773$	$\Phi_0(2.5) = 0.9938$
$t_{0.01}(8) = 3.355$	$t_{0.01}(9) = 3.250$	$t_{0.01}(15) = 2.947$	$t_{0.01}(16) = 2.921$
$\chi^2_{0.005}(8) = 22.0$	$\chi^2_{0.005}(9) = 23.6$	$\chi^2_{0.005}(15) = 32.8$	$\chi^2_{0.005}(16) = 34.3$
$\chi^2_{0.995}(8) = 1.34$	$\chi^2_{0.995}(9) = 1.73$	$\chi^2_{0.995}(15) = 4.60$	$\chi^2_{0.995}(16) = 5.14$